



Developing and Deploying Optimization Applications with AMPL

Speaker Introduction



- **Greg Glockner**

- Vice President of Engineering at Gurobi Optimization
- Manages Gurobi worldwide support
- Global experience in optimization training and consulting
- Winner of the INFORMS Transportation Science dissertation prize



Speaker Introduction

- **Bob Fourer**

- Co-founder of AMPL Optimization
- Professor of Industrial Engineering & Management Sciences, Northwestern University
- Expert in optimization modeling systems & methods
- Co-recipient of the INFORMS Impact Prize for development of algebraic modeling languages
- Created the AMPL modeling language and system with David Gay and Brian Kernighan of Bell Laboratories



- **A wide variety of systems to express optimization models**
 - Gurobi provides APIs for popular programming languages
 - C, C++, C#, Java, Python, MATLAB, R
 - Others provide a variety of higher-level interfaces
 - Spreadsheet optimizers
 - Optimization modeling extensions to programming languages
 - Algebraic modeling systems for optimization
- **This webinar highlights AMPL**
 - A widely used algebraic modeling system
 - Designed for fast development and reliable deployment

Developing and Deploying Optimization Applications with AMPL

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Gurobi Webinar

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The Optimization Modeling Cycle

Steps

- ❖ Communicate with problem owner
- ❖ Build model
- ❖ Prepare data
- ❖ Generate optimization problem
- ❖ Submit problem to Gurobi solver
- ❖ Report & analyze results
- ❖ ***Repeat!***

Goals for optimization software

- ❖ Do this quickly and reliably
- ❖ Get results before client loses interest
- ❖ ***Deploy for application***

Optimization Modeling Languages

Two forms of an optimization problem

- ❖ Modeler's form
 - * High-level description, easy for people to work with
- ❖ Algorithm's form
 - * Explicit data structure, easy for solvers to compute with

Idea of a modeling language

- ❖ ***A computer-readable modeler's form***
 - * You write optimization problems in a modeling language
 - * Computers translate to algorithm's form for solution

Reasons to consider a modeling language

- ❖ Faster modeling cycles
- ❖ More reliable modeling
- ❖ More maintainable applications

Algebraic Modeling Languages

Formulation concept

- ❖ Define data in terms of sets & parameters
 - * Analogous to database keys & records
- ❖ Define decision variables
- ❖ Minimize or maximize a function of decision variables
- ❖ Subject to equations or inequalities that constrain the values of the variables

Advantages

- ❖ Familiar
- ❖ Powerful
- ❖ Proven



Features

- ❖ Algebraic modeling language
- ❖ Built specially for optimization

Design goals

- ❖ Powerful, general expressions
- ❖ Natural, easy-to-learn modeling principles
- ❖ Efficient processing that scales well with problem size

A Note on Performance



- **Frequent customer question:**
 - "What's the best language for building my optimization model?"
- **Underlying questions:**
 - Which is easiest to use?
 - Which fits best with my application?
 - *Which gives the best performance?*

A Simple Example

- **Let's build a simple network-flow model**
 - 100K nodes and 1M edges
 - Resulting model is a continuous linear program
- **Consider time required to**
 - Generate the LP and send it to Gurobi
 - Solve the LP in Gurobi
 - Average from 5 trials on Xeon E3-1240 (3.40 GHz)

```
for (j = 0; j < nedges; j++) {  
    ind[0] = s[j];  
    ind[1] = t[j];  
    val[0] = -1;  
    val[1] = 1;  
  
    error = GRBaddvar(model, 2, ind, val, 0, 0, (double)c[j], GRB_CONTINUOUS, NULL);  
    if (error) goto QUIT;  
}
```

- **Runtime to build model: 0.30 sec**

```
for i in range(N):  
    incoming = sum(x[e] for e in edges.select('*', i))  
    outgoing = sum(x[e] for e in edges.select(i, '*'))  
    m.addConstr(incoming == outgoing)
```

- **Runtime to build model: 6.62 sec**

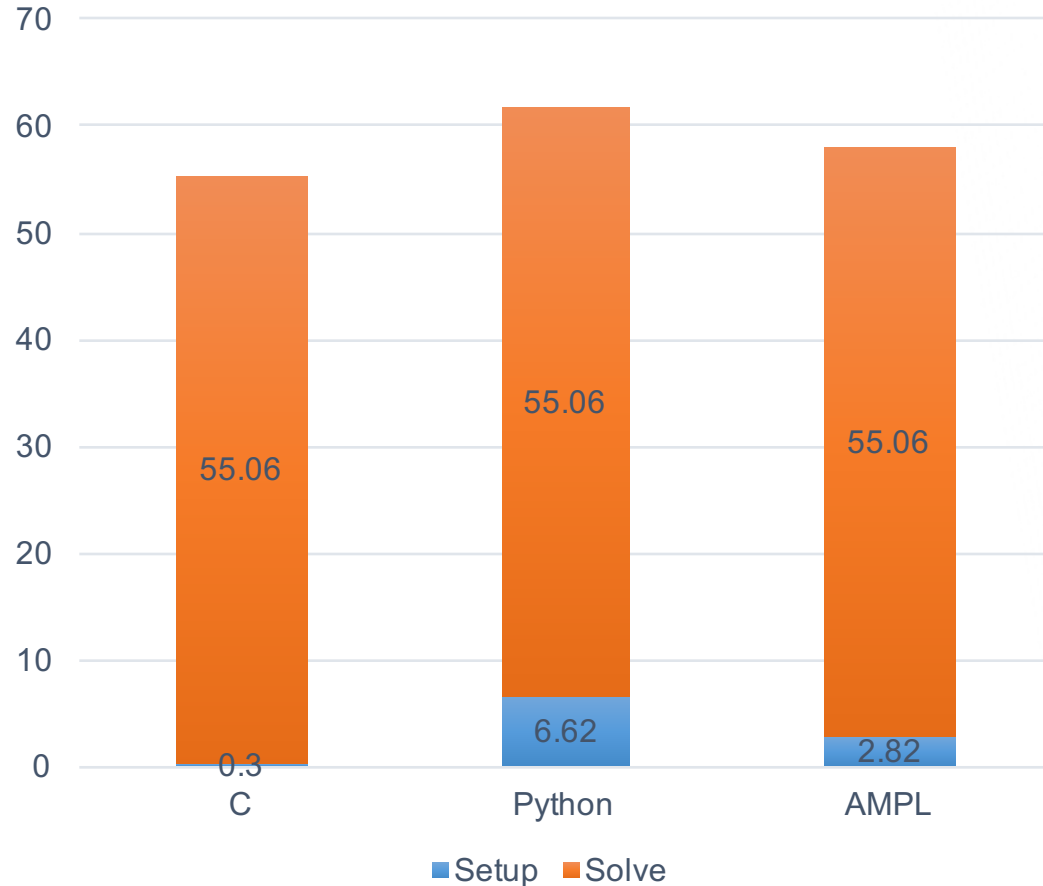
```
subject to Balance {i in NODES diff {0,n-1}}:  
    sum {(j,i) in ARCS} Flow[j,i] = sum {(i,j) in ARCS} Flow[i,j];
```

- **Runtime to build model: 2.81 sec**

Should Performance Guide Language Choice?



Seconds for sample LP



- **Language features should be top priority**
 - Ease of development, deployment and maintenance
 - How well does language fit the needs of the application
- **Performance should be much lower in priority**
 - Time and memory for model setup should be much less than for solving



Features

- ❖ Algebraic modeling language
- ❖ Built specially for optimization
- ❖ Designed to support many solvers

Design goals

- ❖ Powerful, general expressions
- ❖ Natural, easy-to-learn modeling principles
- ❖ Efficient processing that scales well with problem size

3 ways to use . . .

3 Ways to Use AMPL

Command language

- ❖ Browse results & debug model interactively
- ❖ Make changes and re-run

Scripting language

- ❖ Bring the programmer to the modeling language

Programming interface (API)

- ❖ Bring the modeling language to the programmer

Series of Examples

Roll cutting model

- ❖ Solution via command language
- ❖ Tradeoff analysis via scripting

Roll cutting by pattern enumeration

- ❖ via scripting
- ❖ via API

Roll cutting by pattern generation

- ❖ via scripting
- ❖ via API

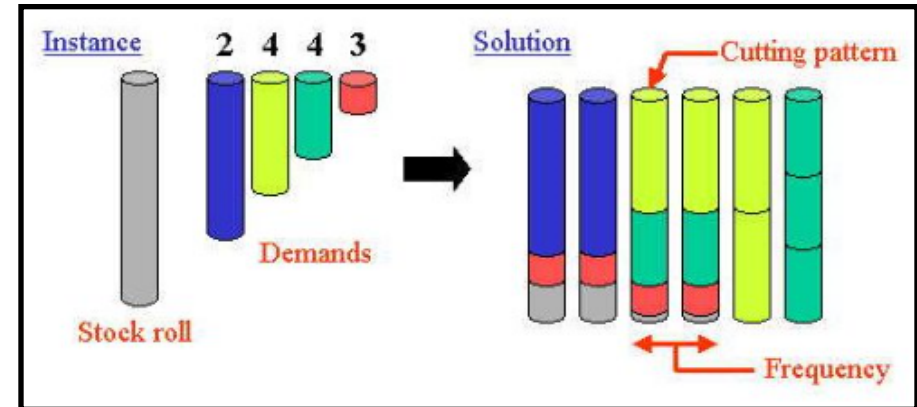
Roll Cutting Problem

Motivation

- ❖ Fill orders for rolls of various widths
 - * by cutting stock rolls of fixed width
 - * using a variety of cutting patterns

Optimization model

- ❖ Decision variables
 - * number of raw rolls to cut according to each pattern
- ❖ Objective
 - * minimize number of raw rolls used
- ❖ Constraints
 - * meet demands for each ordered width



Roll cutting

Algebraic Formulation

Given

W set of ordered widths

n number of patterns considered

and

a_{ij} occurrences of width i in pattern j , for each $i \in W$ and $j = 1, \dots, n$

b_i orders for width i , for each $i \in W$

Roll cutting

Mathematical Formulation (*cont'd*)

Determine

X_j number of rolls to cut using pattern j ,
for each $j = 1, \dots, n$

to minimize

$$\sum_{j=1}^n X_j$$

total number of rolls cut

subject to

$$\sum_{j=1}^n a_{ij} X_j \geq b_i, \text{ for all } i \in W$$

number of rolls of width i cut
must be at least the number ordered

AMPL Formulation

Symbolic model

```
set WIDTHS;  
param orders {WIDTHS} > 0;  
param nPAT integer >= 0;  
param nbr {WIDTHS,1..nPAT} integer >= 0;  
  
var Cut {1..nPAT} integer >= 0;  
  
minimize Number:  
    sum {j in 1..nPAT} Cut[j];  
  
subject to Fulfill {i in WIDTHS}:  
    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

$$\sum_{j=1}^n a_{ij} X_j \geq b_i, \text{ for all } i \in W$$

AMPL Formulation (*cont'd*)

Explicit data (independent of model)

```
param: WIDTHS: orders :=
    6.77    10
    7.56    40
    17.46   33
    18.76   10 ;

param nPAT := 9 ;

param nbr: 1 2 3 4 5 6 7 8 9 :=
    6.77  0 1 1 0 3 2 0 1 4
    7.56  1 0 2 1 1 4 6 5 2
    17.46 0 1 0 2 1 0 1 1 1
    18.76 3 2 2 1 1 1 0 0 0 ;
```

Command Language

Model + data = problem instance to be solved

```
AMPL> model cut.mod;
AMPL> data cut.dat;
AMPL> option solver gurobi;
AMPL> solve;
Gurobi 6.5.0: optimal solution; objective 20
3 simplex iterations
AMPL> option omit_zero_rows 1;
AMPL> option display_1col 0;
AMPL> display Cut;
4 13    7 4    9 3
```

Command Language (*cont'd*)

Results available for browsing

```
ampl: display {j in 1..nPAT, i in WIDTHS: Cut[j] > 0} nbr[i,j];
:          4   7   9   :=                                # patterns used
6.77      0   0   4
7.56      1   6   2
17.46     2   1   1
18.76     1   0   0

ampl: display {j in 1..nPAT} sum {i in WIDTHS} i * nbr[i,j];
1 63.84    3 59.41    5 64.09    7 62.82    9 59.66      # sum of widths
2 61.75    4 61.24    6 62.54    8 62.0        # in each pattern

ampl: display Fulfill.slack;
6.77  2                                # overruns
7.56  3
17.46 0
18.76 3
```

Revision 1: An Alternate Objective

Symbolic model

```
param roll_width > 0;
set WIDTHS;
param orders {WIDTHS} > 0;
param nPAT integer >= 0;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
    sum {j in 1..nPAT} Cut[j];
minimize Waste:
    sum {j in 1..nPAT}
        Cut[j] * (roll_width - sum {i in WIDTHS} i * nbr[i,j]);

subj to Fulfill {i in WIDTHS}:
    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Revision 1 (*cont'd*)

Explicit data

```
param roll_width := 64.5;
param: WIDTHS: orders :=
    6.77    10
    7.56    40
    17.46   33
    18.76   10 ;

param nPAT := 9 ;

param nbr: 1 2 3 4 5 6 7 8 9 :=
    6.77  0 1 1 0 3 2 0 1 4
    7.56  1 0 2 1 1 4 6 5 2
    17.46 0 1 0 2 1 0 1 1 1
    18.76 3 2 2 1 1 1 0 0 0 ;
```

Revision 1 (*cont'd*)

Solutions

```
AMPL: model cutRev1.mod;
AMPL: data cutRev1.dat;

AMPL: objective Number; solve;
Gurobi 6.5.0: optimal solution; objective 20
3 simplex iterations
AMPL: display Number, Waste;
Number = 20
Waste = 63.62

AMPL: objective Waste; solve;
Gurobi 6.5.0: optimal solution; objective 15.62
2 simplex iterations
AMPL: display Number, Waste;
Number = 35
Waste = 15.62
```

Revision 2: Limit on Overruns

Symbolic model

```
param roll_width > 0;  
param over_lim integer >= 0;  
  
set WIDTHS;  
param orders {WIDTHS} > 0;  
  
param nPAT integer >= 0;  
param nbr {WIDTHS,1..nPAT} integer >= 0;  
  
var Cut {1..nPAT} integer >= 0;  
  
...  
  
subj to Fulfill {i in WIDTHS}:  
    orders[i] <= sum {j in 1..nPAT} nbr[i,j] * Cut[j]  
    <= orders[i] + over_lim;
```


Revision 2 (cont'd)

Explicit data

```
param roll_width := 64.5;
param over_lim := 6 ;

param: WIDTHS: orders :=
    6.77    10
    7.56    40
    17.46   33
    18.76   10 ;

param nPAT := 9 ;

param nbr: 1 2 3 4 5 6 7 8 9 :=
    6.77  0 1 1 0 3 2 0 1 4
    7.56  1 0 2 1 1 4 6 5 2
    17.46 0 1 0 2 1 0 1 1 1
    18.76 3 2 2 1 1 1 0 0 0 ;
```

Revision 2 (*cont'd*)

Solutions

```
AMPL: model cutRev2.mod;
AMPL: data cutRev2.dat;

AMPL: objective Number; solve;
Gurobi 6.0.4: optimal solution; objective 20
8 simplex iterations

AMPL: display Number, Waste;
Number = 20
Waste = 63.62

AMPL: objective Waste; solve;
Gurobi 6.0.4: optimal solution; objective 49.16
2 simplex iterations

AMPL: display Number, Waste;
Number = 21
Waste = 49.16
```

Further revisions

Overruns

- ❖ Limit to percentage of amount ordered
- ❖ Limit total extra rolls

Pattern restrictions

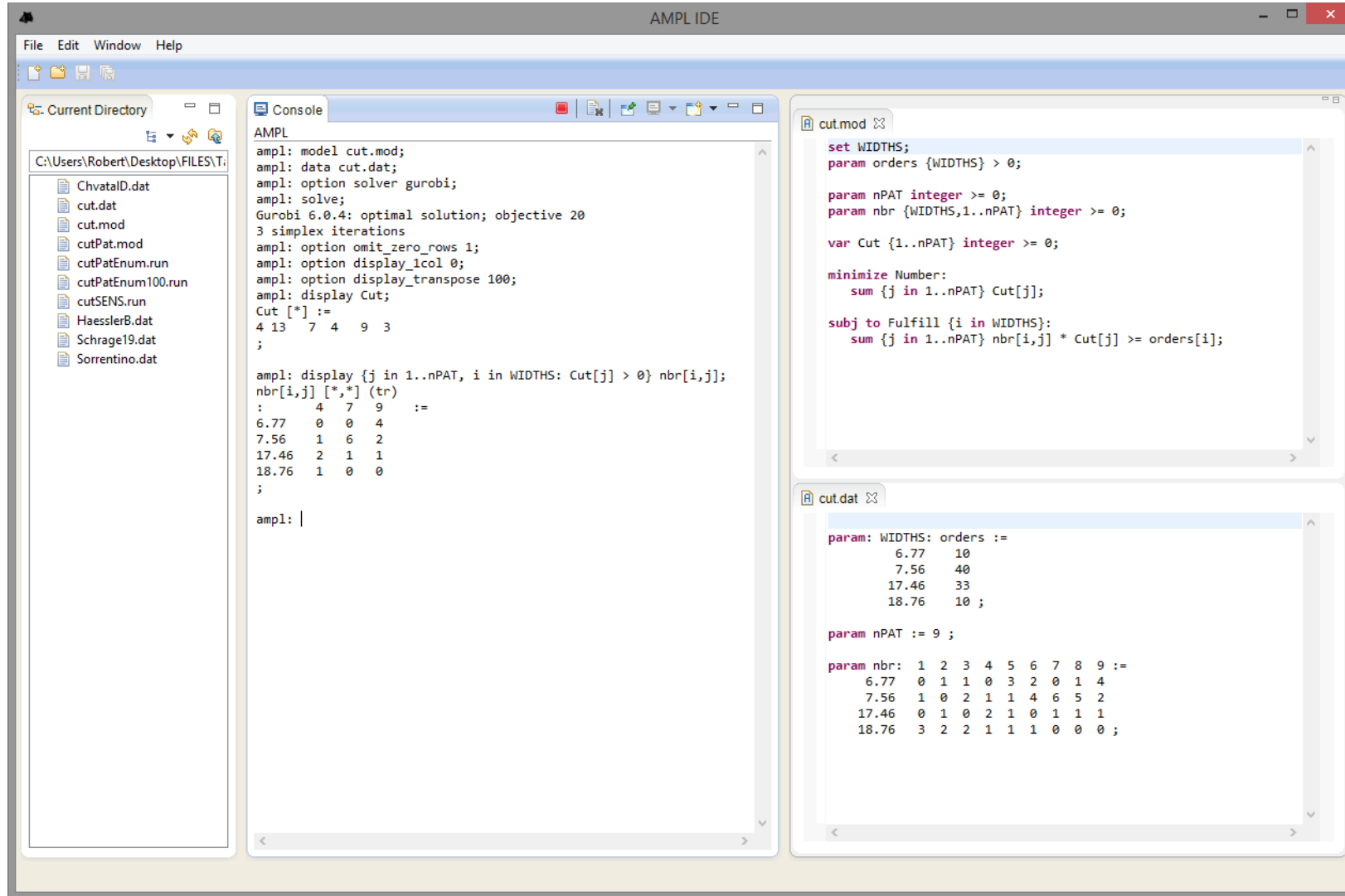
- ❖ Cut at least a specified number of each pattern used
- ❖ Limit the number of patterns used

Costs

- ❖ Account for setups
- ❖ Account for complications of cutting

Anything else you can imagine . . .

IDE for Command Language



Scripting

Bring the programmer to the modeling language

Extend modeling language syntax . . .

- ❖ Algebraic expressions
- ❖ Set indexing expressions
- ❖ Interactive commands

. . . with programming concepts

- ❖ Loops of various kinds
- ❖ If-then and If-then-else conditionals
- ❖ Assignments

Scripting

Examples

Extended analysis

- ❖ Tradeoffs between objectives

Data generation and result processing

- ❖ Cutting *via* pattern enumeration

Optimization schemes

- ❖ Cutting *via* pattern generation

Tradeoffs Between Objectives

Minimize rolls cut

- ❖ Record total rolls cut (low), total waste (high)

Minimize waste

- ❖ Set large overrun limit
- ❖ Record total rolls cut (high), total waste (low)

Explore tradeoffs

- ❖ Reduce overrun limit 1 roll at a time
- ❖ If there is a change in number of rolls cut
 - * record total rolls cut (decreasing)
 - * record total waste (increasing)
- ❖ Stop when no further progress possible
 - * problem becomes infeasible
 - * total rolls cut falls to its minimum

Parametric Analysis (*cont'd*)

Script (setup and initial solve)

```
model cutRev2.mod;
data cutRev2.dat;

set OVER default {} ordered by reversed Integers;

param minNumber;
param minNumWaste;
param minWaste {OVER};
param minWasteNum {OVER};

param prev_number default Infinity;

option solver Gurobi;
option solver_msg 0;

objective Number;
solve >Nul;

let minNumber := Number;
let minNumWaste := Waste;

objective Waste;
```


Parametric Analysis (*cont'd*)

Script (looping and reporting)

```
for {k in over_lim .. 0 by -1} {
  let over_lim := k;
  solve >Nul;
  if solve_result = 'infeasible' then break;
  if Number < prev_number then {
    let OVER := OVER union {k};
    let minWaste[k] := Waste;
    let minWasteNum[k] := Number;
    let prev_number := Number;
  }
  if Number = minNumber then break;
}
printf 'Min%3d rolls with waste%6.2f\n\n', minNumber, minNumWaste;
printf ' Over Waste Number\n';
printf {k in OVER}: '%4d%8.2f%6d\n', k, minWaste[k], minWasteNum[k];
```

Parametric Analysis (*cont'd*)

Script run

```
AMPL: include cutWASTE.run
Min 20 rolls with waste 63.62

Over  Waste  Number
  10   46.72    22
   7   47.89    21
   5   54.76    20

AMPL:
```

Scripting

Cutting *via* Pattern Enumeration

Build the pattern list, then solve

- ❖ Read general model
- ❖ Read data: demands, raw width
- ❖ Compute data: all usable patterns
- ❖ Solve problem instance

Pattern Enumeration

Model

```
param roll_width > 0;
set WIDTHS ordered by reversed Reals;
param orders {WIDTHS} > 0;

param maxPAT integer >= 0;
param nPAT integer >= 0, <= maxPAT;

param nbr {WIDTHS,1..maxPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
    sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:
    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Pattern Enumeration

Data

```
param roll_width := 64.50 ;  
param: WIDTHS: orders :=  
    6.77    10  
    7.56    40  
    17.46   33  
    18.76   10 ;
```

Pattern Enumeration

Script (initialize)

```
model cutPAT.mod;
data Sorrentino.dat;

param curr_sum >= 0;
param curr_width > 0;
param pattern {WIDTHS} integer >= 0;

let maxPAT := 1000000;

let nPAT := 0;
let curr_sum := 0;
let curr_width := first(WIDTHS);
let {w in WIDTHS} pattern[w] := 0;
```

Pattern Enumeration

Script (loop)

```
repeat {
  if curr_sum + curr_width <= roll_width then {
    let pattern[curr_width] := floor((roll_width-curr_sum)/curr_width);
    let curr_sum := curr_sum + pattern[curr_width] * curr_width;
  }
  if curr_width != last(WIDTHS) then
    let curr_width := next(curr_width,WIDTHS);
  else {
    let nPAT := nPAT + 1;
    let {w in WIDTHS} nbr[w,nPAT] := pattern[w];
    let curr_sum := curr_sum - pattern[last(WIDTHS)] * last(WIDTHS);
    let pattern[last(WIDTHS)] := 0;
    let curr_width := min {w in WIDTHS: pattern[w] > 0} w;
    if curr_width < Infinity then {
      let curr_sum := curr_sum - curr_width;
      let pattern[curr_width] := pattern[curr_width] - 1;
      let curr_width := next(curr_width,WIDTHS);
    }
    else break;
  }
}
```

Pattern Enumeration

Script (solve, report)

```
option solver gurobi;
solve;
printf "\n%5i patterns, %3i rolls", nPAT, sum {j in 1..nPAT} Cut[j];
printf "\n\n Cut  ";
printf {j in 1..nPAT: Cut[j] > 0}: "%3i", Cut[j];
printf "\n\n";
for {i in WIDTHS} {
  printf "%7.2f ", i;
  printf {j in 1..nPAT: Cut[j] > 0}: "%3i", nbr[i,j];
  printf "\n";
}
```


Pattern Enumeration

Results

```
ampl: include cutPatEnum.run  
Gurobi 6.5.0: optimal solution; objective 18  
7 simplex iterations  
  
43 patterns, 18 rolls  
Cut      3  1  4 10  
18.76    3  1  0  0  
17.46    0  1  3  2  
7.56     1  3  1  3  
6.77     0  0  0  1
```

Pattern Enumeration

Bigger data

```
param roll_width := 349 ;  
param: WIDTHS: orders :=  
    28.75    7  
    33.75    23  
    34.75    23  
    37.75    31  
    38.75    10  
    39.75    39  
    40.75    58  
    41.75    47  
    42.25    19  
    44.75    13  
    45.75    26 ;
```

Pattern Enumeration

Far more patterns, still fast results

```
ampl: include cutPatEnum.run  
  
Gurobi 6.5.0: optimal solution; objective 34  
291 simplex iterations  
60 branch-and-cut nodes  
54508 patterns, 34 rolls  
  
Cut      8  1  1  2  2  3  1  1  1  1  2  1  4  6  
45.75    3  1  1  0  0  0  0  0  0  0  0  0  0  0  
44.75    1  3  2  0  0  0  0  0  0  0  0  0  0  0  
42.25    0  4  5  4  1  0  0  0  0  0  0  0  0  0  
41.75    4  0  0  0  0  3  2  2  1  1  0  0  0  0  
40.75    0  0  0  3  0  3  2  0  4  3  6  4  3  1  
39.75    0  0  0  0  0  0  0  4  0  0  0  2  4  3  
38.75    0  0  0  0  0  0  3  1  1  3  0  2  0  0  
37.75    0  0  0  0  0  0  0  0  1  0  0  0  0  5  
34.75    0  0  0  0  8  0  0  0  0  1  3  0  0  0  
33.75    0  0  0  0  0  3  2  2  2  1  0  0  2  0  
28.75    0  0  0  2  1  0  0  0  0  0  0  1  0  0
```

Cutting *via* Pattern Generation

Generate the pattern list by a series of solves

- ❖ Solve LP relaxation using subset of patterns
- ❖ Add “most promising” pattern to the subset
 - * Minimize reduced cost given dual values
 - * Equivalent to a one-constraint “knapsack” optimization problem
- ❖ Iterate as long as there are promising patterns
 - * Stop when minimum reduced cost is zero
- ❖ Solve IP using all patterns found

Pattern Generation

Cutting model

```
set WIDTHS ordered by reversed Reals;
param orders {WIDTHS} > 0;

param nPAT integer >= 0, <= maxPAT;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
    sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:
    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Pattern Generation

Knapsack model

```
param roll_width > 0;
param price {WIDTHS} default 0.0;

var Use {WIDTHS} integer >= 0;

minimize Reduced_Cost:
    1 - sum {i in WIDTHS} price[i] * Use[i];

subj to Width_Limit:
    sum {i in WIDTHS} i * Use[i] <= roll_width;
```

Pattern Generation

Script (problems, initial patterns)

```
model cutPatGen.mod;
data Sorrentino.dat;

problem Cutting_Opt: Cut, Number, Fill;
  option relax_integrality 1;
  option presolve 0;

problem Pattern_Gen: Use, Reduced_Cost, Width_Limit;
  option relax_integrality 0;
  option presolve 1;

let nPAT := 0;
for {i in WIDTHS} {
  let nPAT := nPAT + 1;
  let nbr[i,nPAT] := floor (roll_width/i);
  let {i2 in WIDTHS: i2 <> i} nbr[i2,nPAT] := 0;
};
```

Pattern Generation

Script (generation loop)

```
repeat {
  solve Cutting_Opt;
  let {i in WIDTHS} price[i] := Fill[i].dual;
  solve Pattern_Gen;
  printf "\n%7.2f%11.2e ", Number, Reduced_Cost;
  if Reduced_Cost < -0.00001 then {
    let nPAT := nPAT + 1;
    let {i in WIDTHS} nbr[i,nPAT] := Use[i];
  }
  else break;
  for {i in WIDTHS} printf "%3i", Use[i];
};
```


Pattern Generation

Script (final integer solution)

```
option Cutting_Opt.relax_integrality 0;
option Cutting_Opt.presolve 10;
solve Cutting_Opt;

if Cutting_Opt.result = "infeasible" then
    printf "\n*** No feasible integer solution ***\n\n";
else {
    printf "Best integer: %3i rolls\n\n", sum {j in 1..nPAT} Cut[j];
    for {j in 1..nPAT: Cut[j] > 0} {
        printf "%3i of:", Cut[j];
        printf {i in WIDTHS: nbr[i,j] > 0}: "%3i x %6.3f", nbr[i,j], i;
        printf "\n";
    }
    printf "\nWASTE = %5.2f%%\n\n",
        100 * (1 - (sum {i in WIDTHS} i * orders[i]) / (roll_width * Number));
}
```

Pattern Generation

Results (relaxation)

```
ampl: include cutpatgen.run

20.44 -1.53e-01 1 3 2 0
18.78 -1.11e-01 0 1 3 0
18.37 -1.25e-01 0 1 0 3
17.96 -4.17e-02 0 6 0 1
17.94 -1.00e-06

Optimal relaxation: 17.9412 rolls

10.0000 of: 1 x 6.770 3 x 7.560 2 x 17.460
4.3333 of: 1 x 7.560 3 x 17.460
3.1961 of: 1 x 7.560 3 x 18.760
0.4118 of: 6 x 7.560 1 x 18.760

WASTE = 2.02%
```

Pattern Generation

Results (integer)

Rounded up to integer: 20 rolls				
Cut	10	5	4	1
6.77	1	0	0	0
7.56	3	1	1	6
17.46	2	3	0	0
18.76	0	0	3	1
WASTE = 12.10%				
Best integer: 19 rolls				
Cut	10	5	3	1
6.77	1	0	0	0
7.56	3	1	1	6
17.46	2	3	0	0
18.76	0	0	3	1
WASTE = 7.48%				

Integration and Deployment

Your system

- ❖ writes data files
- ❖ Invokes AMPL with script name

AMPL's script

- ❖ reads the data files
- ❖ processes data, generates problems, invokes solvers
- ❖ writes result files

Your system

- ❖ reads the result files

*... multi-file implementations with
hundreds of statements, millions of statements executed*

Limitations

Script statements can be slow

- ❖ Interpreted, not compiled
- ❖ Must support very general set & data structures

Script programming constructs are limited

- ❖ Based on a declarative language
- ❖ Not object-oriented

Scripts are stand-alone

- ❖ Must close AMPL environment before returning to system

So . . .

APIs (application programming interfaces)

Bring the modeling language to the programmer

- ❖ Data and result management in a general-purpose programming language
- ❖ Modeling and solving through calls to an active AMPL process

Cutting Revisited

Hybrid approach

- ❖ Supervision in a general programming language
 - * Data preparation
 - * Pattern enumeration or generation
 - * Result reporting
- ❖ Model & solving in AMPL

Example: Pattern enumeration using MATLAB

- ❖ AMPL entities
- ❖ objects
- ❖ methods for working with AMPL
- ❖ functions

AMPL Model File

Basic pattern-cutting model

```
param nPatterns integer > 0;
set PATTERNS = 1..nPatterns; # patterns
set WIDTHS; # finished widths
param order {WIDTHS} >= 0; # rolls of width j ordered
param overrun; # permitted overrun on any width
param rolls {WIDTHS,PATTERNS} >= 0; # rolls of width i in pattern j

var Cut {PATTERNS} integer >= 0; # raw rolls to cut in each pattern

minimize TotalRawRolls: sum {p in PATTERNS} Cut[p];

subject to FinishedRollLimits {w in WIDTHS}:
    order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;
```


Pattern Enumeration in MATLAB

Load & generate data, set up AMPL model

```
function cuttingEnum(dataFile)

% Get data from .mat file: roll_width, overrun, widths, orders
load(dataFile);

% Generate pattern matrix
[widthsDec,ind] = sort(widths,'descend');
patmat = patternEnum(roll_width,widthsDec);
patmat(:,ind) = patmat;

% Initialize and load cutting-stock model from file
AMPL = AMPL();
AMPL.read('cut.mod');
```

Pattern Enumeration in MATLAB

Send data to AMPL

```
% Send scalar values
AMPL.getParameter('overrun').setValues(overrun);
AMPL.getParameter('nPatterns').setValues(length(patmat));

% Send order vector
WidthOrder = DataFrame(1, 'WIDTHS', 'order');
WidthOrder.setColumn('WIDTHS', num2cell(widths));
WidthOrder.setColumn('order', orders);
AMPL.setData(WidthOrder, 'WIDTHS');

% Send pattern matrix
AllPatterns = DataFrame(2, 'WIDTHS', 'PATTERNS', 'rolls');
AllPatterns.setMatrix('patmat', num2cell(widths), num2cell(1:length(patmat)));
AMPL.setData(AllPatterns)
```

Pattern Enumeration in MATLAB

Solve and report

```
% Solve
ampl.setOption('solver' , 'gurobi');
ampl.solve

% Retrieve solution
CuttingPlan = ampl.getVariable('Cut').getValues();
cutvec = CuttingPlan.getColumnAsDoubles('val');

% Display solution
cuttingPlot (roll_width, widths, patmat(cutvec>0,:), cutvec(cutvec>0))
```

Pattern Enumeration in MATLAB

Enumeration routine

```
function patmat = patternEnum(rollwidth,widths)
if length(widths) == 1
    patmat = floor(rollwidth/widths(1));
else
    patmat = [];
    for n = floor(rollwidth/widths(1)):-1:0
        patnew = patternEnum (rollwidth-n*widths(1), widths(2:end));
        patmat = [patmat; n*ones(size(patnew,1),1) patnew];
    end
end
```

Pattern Enumeration in MATLAB

Plotting routine

```
function cuttingPlot (roll_width,widths,patmat,cutvec)
plotmat = zeros(length(cutvec),sum(max(patmat)));
colors = jet(length(widths));
plotpos = 0;
for j = 1:length(widths)
    for i = 1:length(cutvec)
        plotmat(i,plotpos+1:plotpos+patmat(i,j)) = widths(j);
    end
    for i = 1:max(patmat(:,j))
        colormat(plotpos+i,:) = colors(j,:);
    end
    plotpos = plotpos + max(patmat(:,j));
end
colormap(colormat); shading faceted
h = barh(plotmat,'stacked');

set (h, 'edgecolor','black'); set(gca,'YTickLabel',num2cell(cutvec))
xlim([0,roll_width]); ylim([0,numel(get(gca,'YTick'))+1])
```

Pattern Enumeration in MATLAB

The image shows the MATLAB R2016a environment. The Command Window displays the following text:

```
>> cuttingEnum('Sorrentino')
Gurobi 6.5.0: optimal solution; objective 18
8 simplex iterations
fx >> |
```

The Figure window, titled 'Figure 1', shows a horizontal bar chart with the following data series:

Category	Segment 1 (Cyan)	Segment 2 (Yellow)	Segment 3 (Red)
10	0-8	8-30	30-65
4	0-8	8-42	42-60
1	0-8	8-25	25-60
1	0-8	8-25	25-65
2	0-8	8-25	25-65

Other Examples

Pattern enumeration in other languages

- ❖ Java, C++, MATLAB currently available
- ❖ Python, R, .NET (VB & C#) under development

Pattern generation, looping over two solves

- ❖ Knapsack solver to find a new pattern
 - * freely available routine using dynamic programming approach
- ❖ Linear solver to find new solution & dual values
 - * Gurobi solver applied to an AMPL model

AMPL Summary

Prototyping and development

- ❖ Interactive modeling interface
 - * Plain command window
 - * Multi-window IDE
- ❖ Scripting facility
 - * Execute AMPL commands
 - * Invoke user-defined AMPL functions
 - * Run external programs

Deployment

- ❖ Interaction with scripts through files
- ❖ Control of AMPL through API in popular programming languages

Trying Gurobi and AMPL for yourself



- *If you already have Gurobi but not AMPL...*
 - Contact sales@gurobi.com and request a free 30-day evaluation license of AMPL
- *If you already have AMPL but not Gurobi...*
 - Commercial: Contact sales@ampl.com and request a free 30-day evaluation license of Gurobi
 - or -
 - Academic: Follow the instructions on: <http://www.gurobi.com/academia/for-universities>
- *If you are new to both AMPL and Gurobi...*
 - Contact either sales@gurobi.com or sales@ampl.com and you will be helped