# Parallel and Distributed Optimization with Gurobi Optimizer

GUROBI OPTIMIZATION

# Our Presenter

**Dr. Tobias Achterberg**
Developer, Gurobi Optimization

# Parallel & Distributed Optimization

# Terminology for this presentation

## Parallel computation

- One computer
  - Multiple processor cores
  - 1 or more processor sockets
- Part of Gurobi throughout our history
  - MIP branch-and-cut
  - Barrier for LP, QP and SOCP
  - Concurrent optimization

## Distributed computation

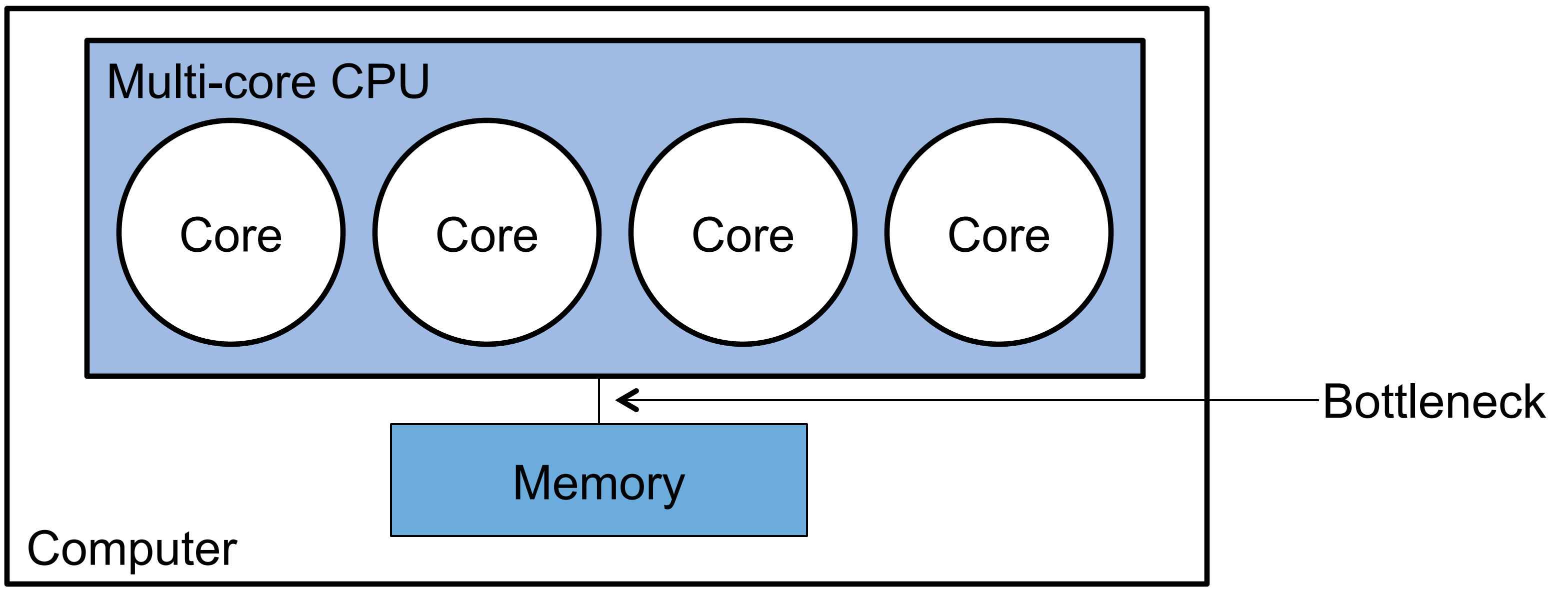
- Multiple computers, linked via a network
- Relatively new feature
- Each independent computer can do parallel computation!

# Parallel algorithms and hardware

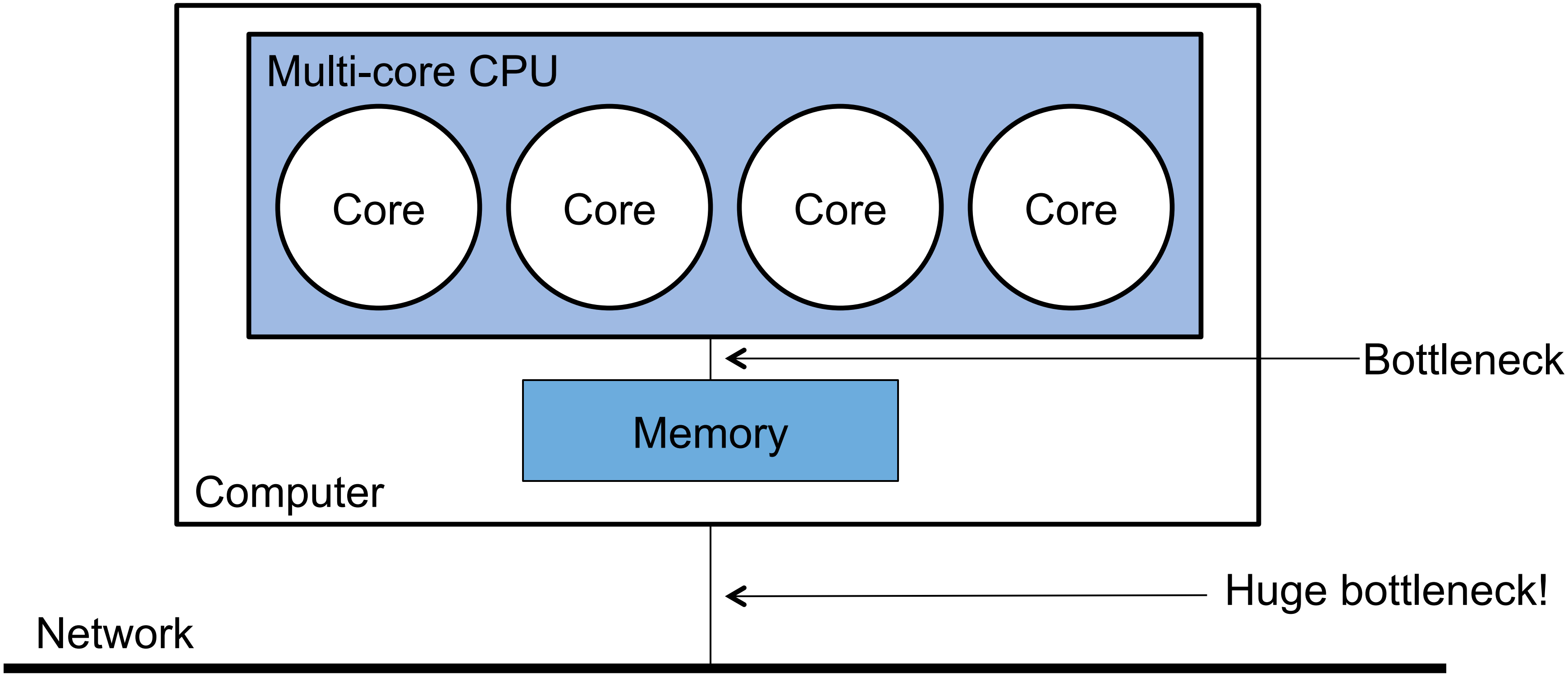
- Parallel algorithms must be designed around hardware
  - What work should be done in parallel
  - How much communication is required
  - How long will communication take

- Goal: Make best use of available processor cores

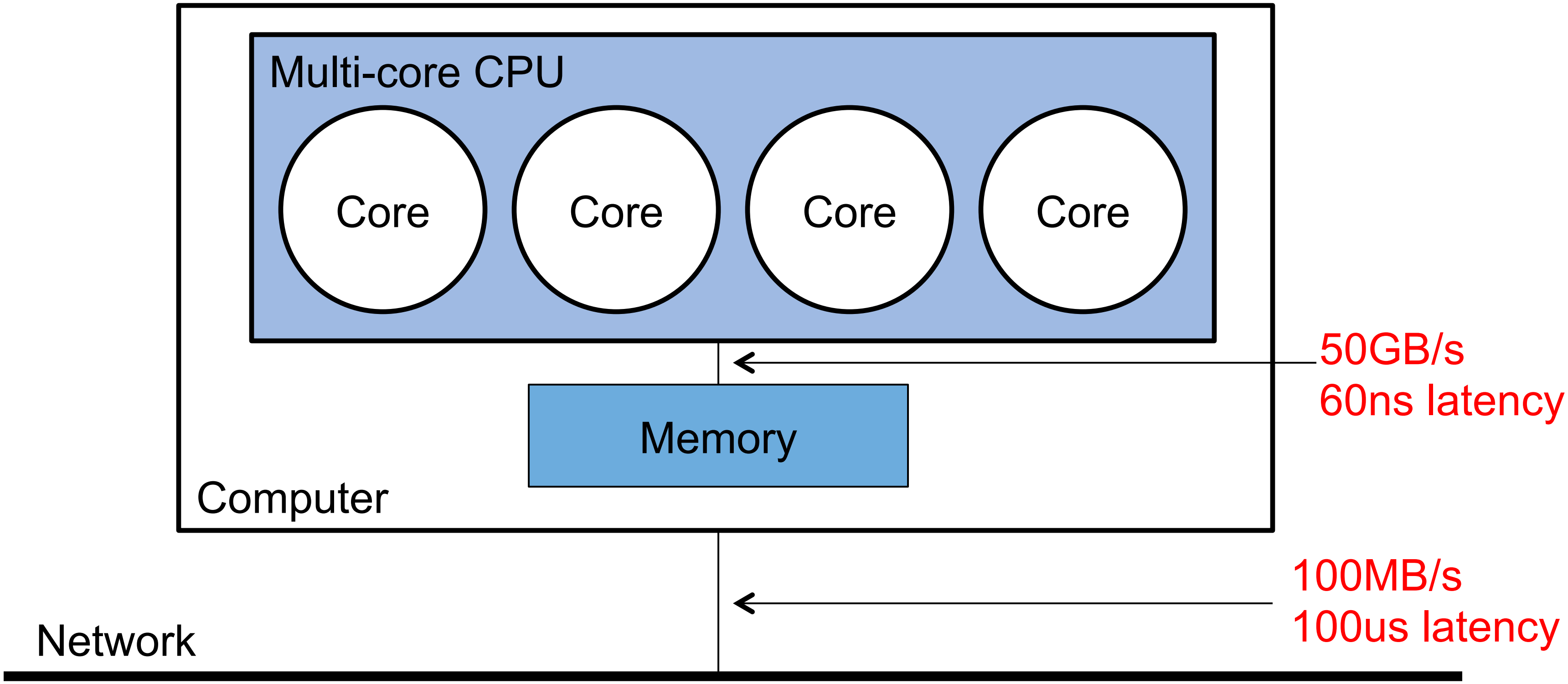
# Multi-Core Hardware



# Distributed Computing



# How Slow Is Communication?




- Network is ~1000x slower than memory
  - Faster on a supercomputer, but still relatively slow

# Distributed Algorithms in Gurobi 6.0

- 3 distributed algorithms in version 6.0
  - Distributed tuning
  - Distributed concurrent
    - LP (new in 6.0)
    - MIP
  - Distributed MIP (new in 6.0)

# Distributed Tuning

- Tuning:
  - MIP has lots of parameters
  - Tuning performs test runs to find better settings
- Independent solves are obvious candidate for parallelism
- Distributed tuning a clear win
  - 10x faster on 10 machines
- Hard to go back once you have tried it

# Concurrent Optimization

# Concurrent Optimization

- Run different algorithms/strategies on different machines/cores
  - First one that finishes wins
- Nearly ideal for distributed optimization
  - Communication:
    - Send model to each machine
    - Winner sends solution back
- Concurrent LP:
  - Different algorithms:
    - Primal simplex/dual simplex/barrier
- Concurrent MIP:
  - Different strategies
  - Default: vary the seed used to break ties
- Easy to customize via concurrent environments

# MIPLIB 2010 Testset

- MIPLIB 2010 test set…
  - Set of 361 mixed-integer programming models
  - Collected by academic/industrial committee
- MIPLIB 2010 benchmark test set…
  - Subset of the full set – 87 of the 361 models
    - Those that were solvable by 2010 codes
    - (Solvable set now includes 206 of the 361 models)
- Notes:
  - Definitely not intended as a high-performance computing test set
    - More than 2/3 solve in less than 100s
    - 8 models solve at the root node
    - ~1/3 solve in fewer than 1000 nodes

# Distributed Concurrent MIP

- Results on MIPLIB benchmark set (>1.00x means concurrent MIP is faster):
  - 4 machines vs 1 machine:

| Runtime | Wins | Losses | Speedup |
|---|---|---|---|
| >1s | 38 | 20 | 1.26x |
| >100s | 17 | 3 | 1.50x |

  - 16 machines vs 1 machine:

| Runtime | Wins | Losses | Speedup |
|---|---|---|---|
| >1s | 54 | 19 | 1.40x |
| >100s | 26 | 1 | 2.00x |

# Customizing Concurrent

- Easy to choose your own settings:
  - Example – 2 concurrent MIP solves:
    - Aggressive cuts on one machine
    - Aggressive heuristics on second machine
    - Java example

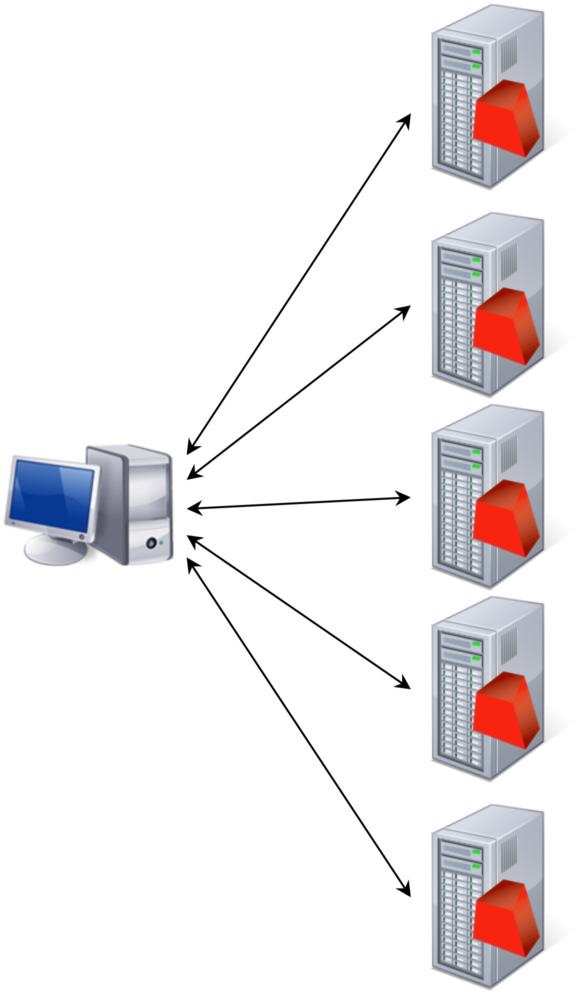
```
GRBEnv env0 = model.getConcurrentEnv(0);
GRBEnv env1 = model.getConcurrentEnv(1);
env0.set(GRB.IntParam.Cuts, 2);
env1.set(GRB.DoubleParam.Heuristics, 0.2);
model.optimize();
model.discardConcurrentEnvs();
```

    - Also supported in C++, .NET, Python and C

# Distributed MIP

# Distributed MIP Architecture

- Manager–worker paradigm
- Manager
  - Send model to all workers
  - Track dual bound and worker node counts
  - Rebalance search tree to put useful load on all workers
  - Distribute feasible solutions
- Workers
  - Solve MIP nodes
  - Report status and feasible solutions
- Synchronized deterministically



# Distributed MIP Phases

- Racing ramp-up phase
  - Distributed concurrent MIP
    - Solve same problem individually on each worker, using different parameter settings
    - Stop when problem is solved or “enough” nodes are explored
    - Choose a “winner” – worker that made the most progress

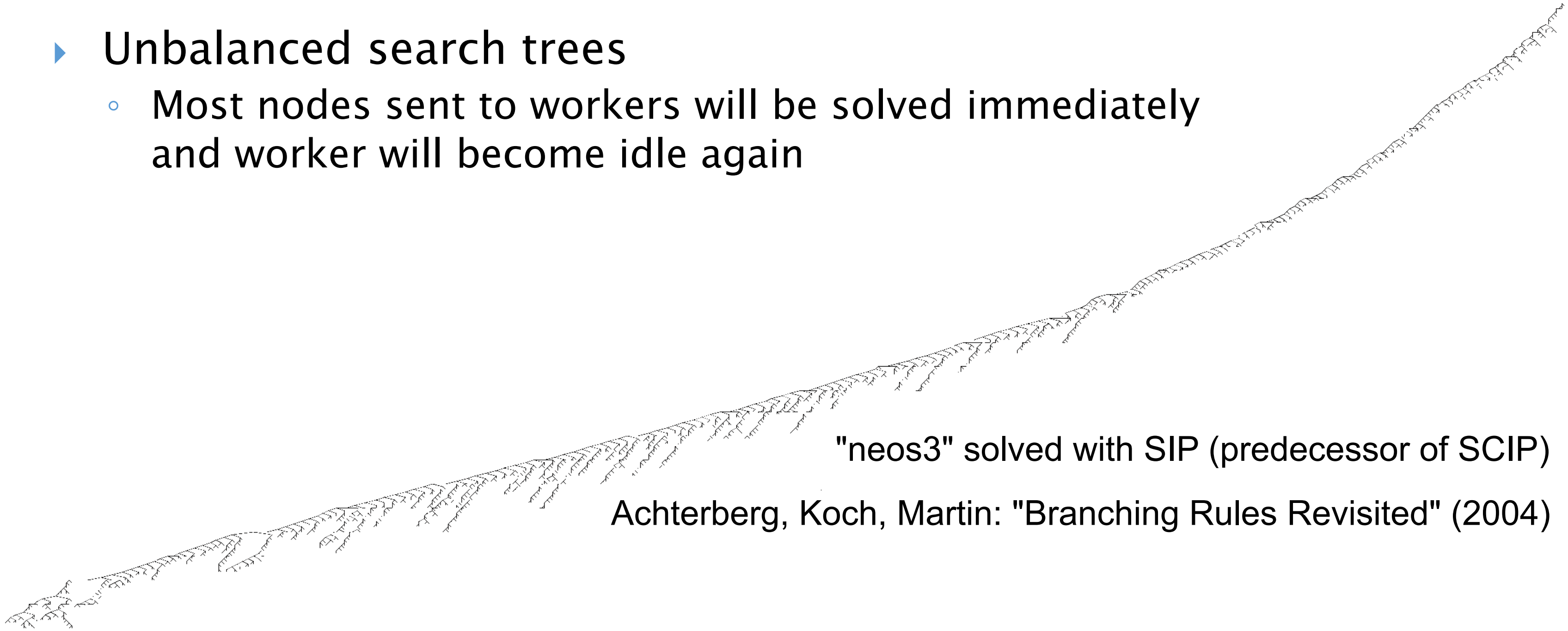
- Main phase
  - Discard all worker trees except the winner's
  - Collect active nodes from winner, distribute them among now idle workers
  - Periodically synchronize to rebalance load

# Bad Cases for Distributed MIP

- Easy problems
  - Why bother with heavy machinery?

- Small search trees
  - Nothing to gain from parallelism

- Unbalanced search trees
  - Most nodes sent to workers will be solved immediately and worker will become idle again

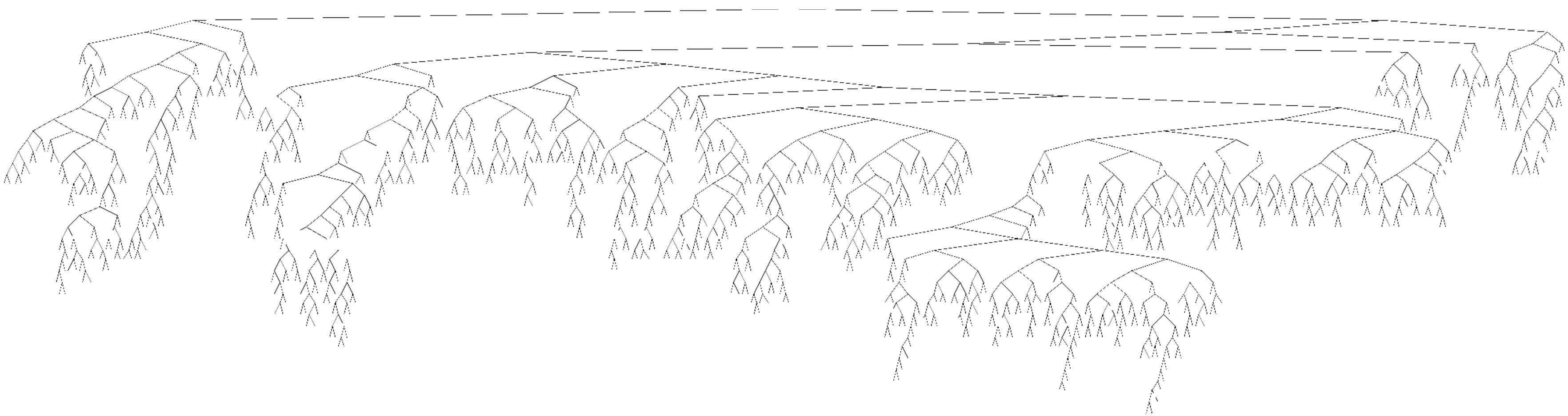


"neos3" solved with SIP (predecessor of SCIP)

Achterberg, Koch, Martin: "Branching Rules Revisited" (2004)

# Good Cases for Distributed MIP

- Large search trees
- Well-balanced search trees
  - Many nodes in frontier lead to large sub-trees



"vpm2" solved with SIP (predecessor of SCIP)

Achterberg, Koch, Martin: "Branching Rules Revisited" (2004)

# Performance

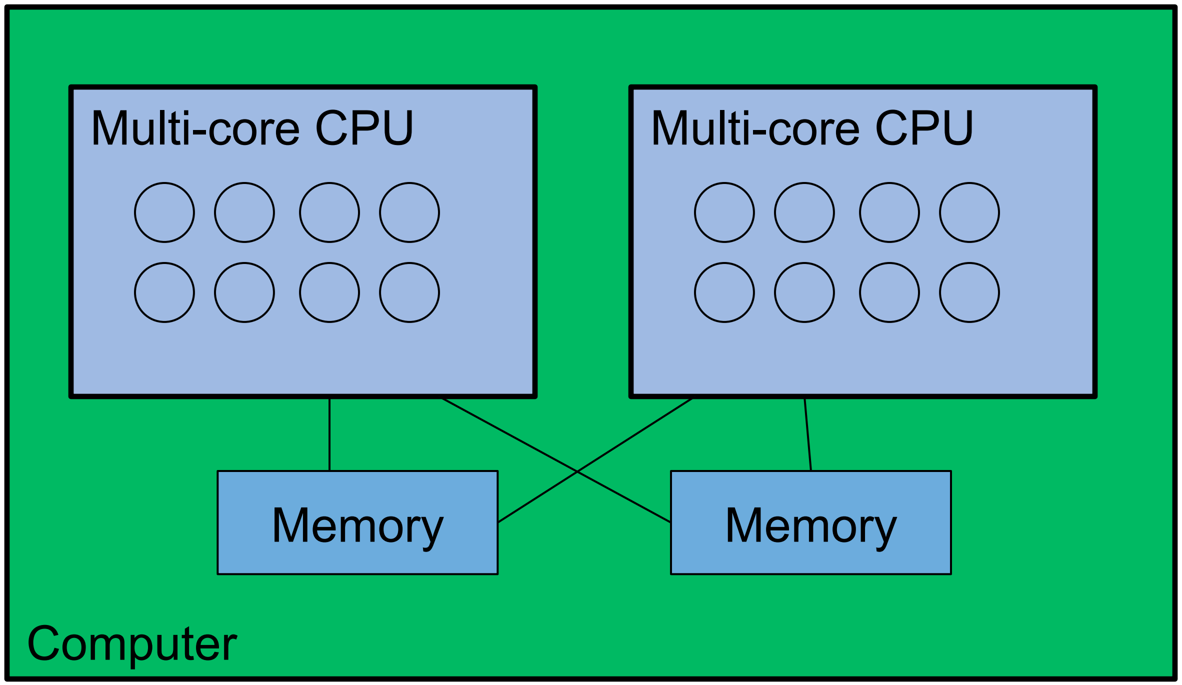
# Three Views of 16 Cores

- Consider three different tests, all using 16 cores:
  - On a 16-core machine:
    - Run the standard parallel code on all 16 cores
    - Run the distributed code on four 4-core subsets
  - On four 4-way machines:
    - Run the distributed code

- Which gives the best results?

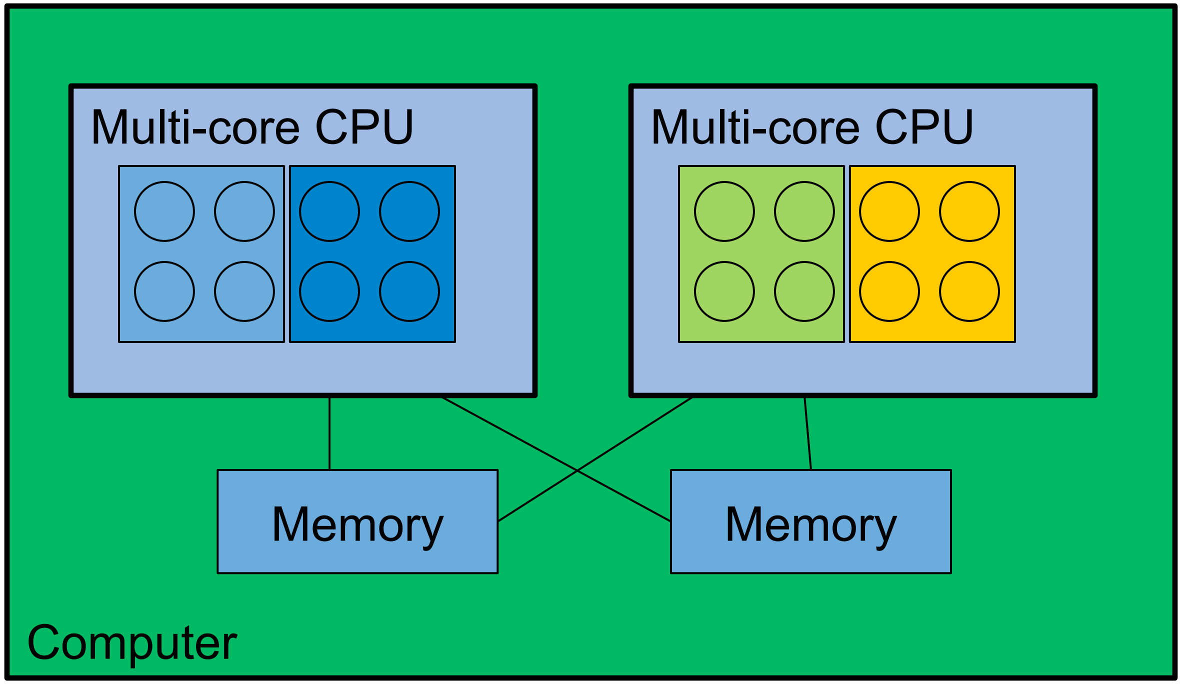
# Parallel MIP on 1 Machine

- Use one 16-core machine:



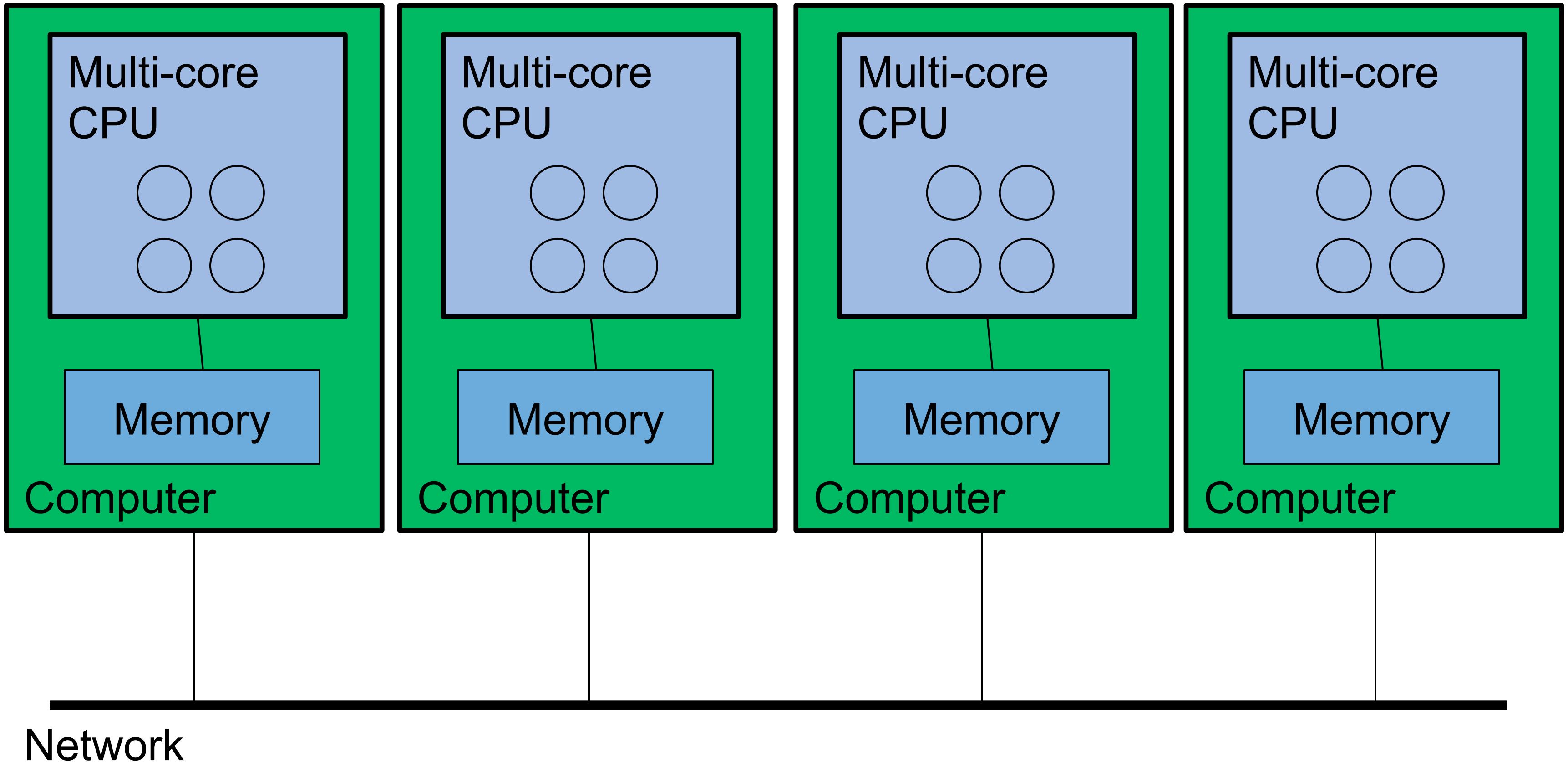
# Distributed MIP on 1 machine

- Treat one 16-core machine as four 4-core machines:



# Distributed MIP on 4 machines

- Use four 4-core machines



# Performance Results

- Using one 16-core machine (MIPLIB 2010, baseline is 4-core run on the same machine)…

| Config | >1s | >100s |
|---|---|---|
| One 16-core | 1.57x | 2.00x |
| Four 4-core | 1.26x | 1.82x |

- Better to run one-machine algorithm on 16 cores than treat the machine as four 4-core machines
  - Degradation isn't large, though

# Performance Results

- Comparing one 16-core machine against four 4-core machines (MIPLIB 2010, baseline is single-machine, 4-core run)…

| Config | >1s | >100s |
|---|---|---|
| One 16-core machine | 1.57x | 2.00x |
| Four 4-core machines | 1.43x | 2.09x |

- Given a choice…
  - Comparable mean speedups
  - Other factors…
    - Cost: four 4-core machines are much cheaper
    - Admin: more work to admin 4 machines

# Distributed Algorithms in 6.0

- MIPLIB 2010 benchmark set
  - Intel Xeon E3-1240v3 (4-core) CPU
  - Compare against 'standard' code on 1 machine

| Machines | >1s | | | >100s | | |
|---|---|---|---|---|---|---|
| | Wins | Losses | Speedup | Wins | Losses | Speedup |
| 2 | 40 | 16 | 1.14x | 20 | 7 | 1.27x |
| 4 | 50 | 17 | 1.43x | 25 | 2 | 2.09x |
| 8 | 53 | 19 | 1.53x | 25 | 2 | 2.87x |
| 16 | 52 | 25 | 1.58x | 25 | 3 | 3.15x |

# Some Big Wins

- Model *seymour*
  - Hard set covering model from MIPLIB 2010
  - 4944 constraints, 1372 (binary) variables, 33K non-zeroes

| Machines | Nodes | Time (s) | Speedup |
|---|---|---|---|
| 1 | 476,642 | 9,267 | – |
| 16 | 1,314,062 | 1,015 | 9.1x |
| 32 | 1,321,048 | 633 | 14.6x |

# Some Big Wins

- Model *a1c1s1*
  - lot sizing model from MIPLIB 2010
  - 3312 constraints, 3648 variables (192 binary), 10k non-zeros

| Machines | Nodes | Time (s) | Speedup |
|---|---|---|---|
| 1 | 3,510,833 | 17,299 | – |
| 49 | 9,761,505 | 1,299 | 13.3x |

# Distributed Concurrent Versus Distributed MIP

- MIPLIB 2010 benchmark set (versus 1 machine run):
  - >1s

| Machines | Concurrent | Distributed |
| --- | --- | --- |
| 4 | 1.26x | 1.43x |
| 16 | 1.40x | 1.58x |

  - >100s

| Machines | Concurrent | Distributed |
| --- | --- | --- |
| 4 | 1.50x | 2.09x |
| 16 | 2.00x | 3.15x |

# Gurobi Distributed MIP

- Makes huge improvements in performance possible

- Mean performance improvements are significant but not huge
  - Some models get big speedups, but many get none
  - Much better than distributed concurrent
  - As effective as adding more cores to one box

- Effectively exploiting parallelism remains:
  - A difficult problem
  - A focus at Gurobi

# How To Use Distributed Algorithms in Gurobi?

# Gurobi Remote Services

- Install Gurobi Remote Services on worker machines
  - No Gurobi license required on workers
  - Machine listens for Distributed Worker requests
- Set a few parameters on manager
  - `ConcurrentJobs=4`
  - `WorkerPool=machine1,machine2,machine3,machine4`
  - No other code changes required
- Manager must be licensed to use distributed algorithms
  - Gurobi Distributed Add-On
    - Enables up to 100 workers

# Integral Part of Product

- Built on top of Gurobi Compute Server
  - Only 1500 lines of C code specific to concurrent/distributed MIP
- Built into the product
  - No special binaries involved

- Bottom line:
  - Changes to MIP solver automatically apply to distributed code too
    - Performance gains in regular MIP also benefit distributed MIP
  - Distributed MIP will evolve with regular MIP

# Licensing

- Commercial
  - Not included – must purchase the distributed option
  - Ask your sales representative for benchmarks or pricing

- Academic
  - Named-user: not included in licenses from Gurobi website
  - Site license: includes distributed parallel algorithms

# Questions

Please use the Question box to ask questions to the speaker

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